DATE: WEDNESDAY 2 OCTOBER 2013

1. PRINT YOUR LAST NAME IN LARGE CAPITAL LETTERS ON THE UPPER RIGHT CORNER OF EACH SHEET TURNED IN.

2. PRINT YOUR FIRST NAME IN CAPITAL LETTERS DIRECTLY UNDERNEATH YOUR LAST NAME ON EACH SHEET TURNED IN.

3. WRITE YOUR CORRECT SECTION NUMBER DIRECTLY UNDER YOUR FIRST NAME.

CIRCLE THE BOLDFACE LETTER INDICATING THE CORRECT ANSWER IN EACH OF THE PROBLEMS BELOW.

4. If \( f(x) = x^5 - 7x^3 + 8x^2 - 9 \), then the derivative of \( f \), denoted \( f' \) is given by \( f'(x) = \)
   
   A. \( 5x^4 - 3x^2 + 2x - 9 \)
   B. \( 5x^4 - 21x^2 + 16x - 9 \)
   C. \( 5x^4 - 21x^2 + 16x \)
   D. \( x^4 - 7x^2 + 8x \)
   E. NONE OF THE ABOVE

CORRECT ANSWER CHOICE: C

\[
f'(x) = 5x^4 - 7 \cdot 3x^2 + 8 \cdot 2x = 5x^4 - 21x^2 + 16x.
\]

5. If \( f(x) = x^4 - 9 \) and \( g(x) = x^3 + 7 \), and if \( h = f - g \), then \( h'(x) = \)
   
   A. \( (4x^3)(3x^2) - 63 \)
   B. \( 4x^3 + 3x^2 \)
   C. \( 4x^3 - 3x^2 \)
   D. \( x^3 - x^2 \)
   E. NONE OF THE ABOVE

CORRECT ANSWER CHOICE: C

\[
h'(x) = f'(x) - g'(x) = 4x^3 - 3x^2.
\]
6. If \( f(x) = x^4 - 9 \) and \( g(x) = x^3 + 7 \), and if \( h = f \cdot g \), then \( h'(x) = \)

A. \((4x^3)(3x^2) - 63\)
B. \((4x^3)(3x^2)\)
C. \((4x^3)(x^3 + 7) + (x^4 - 9)(3x^2)\)
D. \((4x^3 - 9)(x^3 + 7) + (x^4 - 9)(3x^2 + 7)\)
E. NONE OF THE ABOVE

CORRECT ANSWER CHOICE: C

\[
\begin{align*}
h'(x) &= f'(x)g(x) + f(x)g'(x) = (4x^3)(x^3 + 7) + (x^4 - 9)(3x^2)\end{align*}
\]

Give the slope of the tangent line to the graph of each of the following functions at the indicated point.

7. \( f(x) = x^2 \) at the point on the graph of \( f \) where \( x = 3 \).

A. 9
B. 8
C. 7
D. 6
E. NONE OF THE ABOVE

CORRECT ANSWER CHOICE: D

We know \( f'(x) = 2x \), and therefore,

\[
\text{Tangent slope } = f'(3) = 2 \cdot 3 = 6.
\]

8. \( f(x) = 5x^3 - x^{10} \) at the point on the graph of \( f \) where \( x = 1 \).

A. 5
B. -5
C. 15
D. -10
E. NONE OF THE ABOVE

CORRECT ANSWER CHOICE: A

We know \( f'(x) = 15x^2 - 10x^9 \), so

\[
\text{Tangent slope } = f'(1) = 15 - 10 = 5.
\]
9. \( f(x) = g(x)h(x) \) at the point where \( x = 5 \) given that
\[
g(5) = 2, g'(5) = 3, h(5) = 7, \text{ and } h'(5) = 4.
\]

A. 12  
B. 13  
C. 14  
D. 15  
E. NONE OF THE ABOVE

CORRECT ANSWER CHOICE: E

\[
f'(x) = g'(x)h(x) + g(x)h'(x),
\]

so
\[
\text{Tangent slope} = g'(5)h(5) + g(5)h'(5) = (3)(7) + (2)(4) = 21 + 8 = 29.
\]

10. The equation of the tangent line to the graph of the function \( f(x) = x^5 \) at the point where \( x = 1 \) is

A. \( y + 1 = 5(x - 1) \)  
B. \( y = x^5 + (5x^4)(x - 1) \)  
C. \( y = x - 1 \)  
D. \( y + 1 = 5x^4 \)  
E. NONE OF THE ABOVE

CORRECT ANSWER CHOICE: E

We know \( f'(x) = 5x^4 \), therefore

\[
\text{Tangent slope} = f'(1) = 5,
\]

and the tangent line passes through the point \((1, f(1))\), but \( f(1) = 1 \), so the tangent line passes through the point \((1, 1)\) with slope 5 and therefore its equation in point slope form is

\[
y = 1 + 5(x - 1),
\]

which is not equivalent to any of the equation choices so the correct answer choice is E.

COMMENT. You should notice that answer choices B and D are not even linear equations so cannot be equations of lines and in particular cannot be tangent line equations. Equation C does not have slope 5 since in fact it has slope 1 and equation A has the correct slope but does not pass through the point of tangency which is \((1, 1)\).