Math 1210 - Calculus I
Recitation problems for Tuesday September 20 / Thursday September 22.

Directions: Attempt all problems before your recitation meeting. You will go over these problems with your TA during the recitation period. Your weekly quiz may be based on these problems, per your instructor’s discretion.

1. Evaluate the following limits.
   (a) \( \lim_{x \to 7} \frac{3}{x - 7} \)
   (b) \( \lim_{x \to -3} \frac{x + 3}{\sqrt{84 + x} - 9} \)
   (c) \( \lim_{x \to 1} \frac{x^4 - 1}{x^3 - 1} \)

2. On a set of axes, draw the graph of a function \( f(x) \) that has ALL of the following properties.
   - The domain of \( f \) is the set of all real numbers \( x \) such that \( x > 0 \).
   - \( \lim_{x \to 0^+} f(x) = -\infty \)
   - \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = +\infty \)

   (Note: In the book’s notation, \( \lim_{x \to 2^-} f(x) = \lim_{x \nearrow 2} f(x) \), \( \lim_{x \to 2^+} f(x) = \lim_{x \searrow 2} f(x) \), etc.)

3. The floor function \( \text{floor}(x) \) calculates the largest integer less than or equal to \( x \), and the ceiling function \( \text{ceiling}(x) \) calculates the smallest integer greater than or equal to \( x \). Here are some examples:
   - \( \text{floor}(4.5) = 4 \)
   - \( \text{floor}(-2.8) = -3 \)
   - \( \text{floor}(8) = 8 \)
   - \( \text{ceiling}(6.1) = 7 \)
   - \( \text{ceiling}(-1.99) = -1 \)

   Now define a function \( g \) by
   \[ g(x) = \frac{\text{ceiling}(x)}{\text{floor}(x)}. \]

   (a) What is the domain of \( g \)?
   (b) Graph \( g \) on the interval \([-5, 5]\). (You might want to use graph paper for this graph.)
   (c) Calculate the following limits or state that they don’t exist.
   i. \( \lim_{x \to 2} g(x) \)
   ii. \( \lim_{x \to -\frac{3}{2}} g(x) \)
   iii. \( \lim_{x \to 0^-} g(x) \)
   iv. \( \lim_{x \to \frac{3}{2}^+} g(x) \)
   (d) Find a value for \( c \) such that \( \lim_{x \to c} g(x) = \frac{3}{2} \). Explain briefly how you used mathematics to obtain your value for \( c \).